Supply Response to Agricultural Insurance: Risk Reduction and Moral Hazard Effects

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This paper examines the consequences of agricultural insurance for expected supply. The effect of insurance is shown to decompose into a "risk reduction" effect as well as a "moral hazard" effect. The direction and magnitude of these effects depend on the parameters of the insurance contract, producer's risk preferences, and the underlying technology. Two models are considered for this purpose. In the first model, widely employed in the literature, a producer controls only one input. The second model uses a dual approach to extend the results to the case where a producer controls multiple inputs.

Key words: agricultural insurance, moral hazard, risk, supply response.

A long standing issue in the analysis of agricultural insurance is supply response. Because an insurance program alters the probability distribution of farm income, the question is whether and in what manner producers adjust supply in response to this change. The answer is of interest to policy makers in developed and developing countries, although for different reasons. In reviewing the history of crop insurance in the U.S., Kramer writes "Research on price and income stabilization programs has indicated that these programs have had supply response effects. As the crop insurance program becomes a truly national program, similar effects may become evident from crop insurance, complicating the supply control objective of commodity programs" (p. 200). Developing country policy makers, on the other hand, place high priority on expanding agricultural supplies. For them, a positive supply response is not a complication but a strong argument for publicly financed crop insurance programs which remove or minimize the influence of risk on farm-level decision making.[1] In the words of Hazell, Pomareda, and Valdes, "Empirical evaluation of the social costs and returns of publicly subsidized crop insurance requires measurement of the effect of risk reduction on supply responses. . . . It is this risk response effect that leads to the major social gain from crop insurance" (p. 8). The study of supply response is therefore important for an evaluation of agricultural insurance programs and consequently for the design of insurance itself.

In the short run, supplies are altered by changes in levels of variable inputs (input choice) and in the allocations of fixed inputs (e.g., land) between competing agricultural activities (activity choice). Crop insurance could affect both these choices. It is well known that insurance affects the incentives for input use. However, the direction of impact has been analyzed for a limited case only. Previous work has observed that risk averse input decisions coincide with risk-neutral decisions if insurance is complete (i.e., eliminates all risk), actuarially fair and contingent on input use. In this case, the effect of insurance is straightforward. Compared to the no-insurance world, the use of risk increasing inputs increases while that of risk decreasing inputs falls, leading to an increase in expected

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[1] For example, see Hazell, Bassoco, and Arcia for the objectives of policy makers in Mexico and Panama. See also Lloyd and Mauldon for excerpts of the report of the Australian Industries Assistance Commission, which discusses the objectives which should guide government policy concerning agricultural instability.

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income (Ahsan, Ali, and Kurian; Nelson and Loehman). This case assumes that insurance payments are contingent on output realizations as well as input choices, which for reasons explained below, is a limiting assumption. However, if the assumption is maintained, its implication for activity choice is that complete specialization would occur in the higher value riskier activity (e.g., Ahsan, Ali, and Kurian). Such an outcome is confirmed in Hazell, Bassoco, and Arcia who simulate the effect of crop insurance on cropping patterns in a sectoral model for Mexico. They find that an insurance program that is actuarially fair and complete leads to higher expected production levels and a shift towards riskier crops.

The desirability of complete insurance, however, depends (among other things) on whether insurance agencies are able to monitor farm level input use. If input use is not monitored, a producer who is completely insured will have little incentive to apply any inputs at all. Then, it is not at all obvious that insurance would induce greater specialization in the riskier crop. The situation where the insurer is unable to monitor input choices of insured farmers is one of moral hazard. It is recognized as a major problem in the practice and design of agricultural insurance (Chambers, Nelson and Loehman, Ray). The widespread use of deductibles and the lack of complete insurance in real world situations is, in part, due to moral hazard considerations. A realistic understanding of the effects of agricultural insurance must take moral hazard into account. This demands that empirical work on supply response be based on models of input use under moral hazard.

A numerical model of input and activity choice under moral hazard has been constructed by Kaylen, Loehman, and Preckel. The model is simulated for a hypothetical example with particular specifications of utility functions, production functions as well as insurance programs. While such models are necessary for delivering quantitative predictions, it is equally important that the qualitative predictions of these models be robust to small changes in numerical specifications. In other words, are there any theoretical predictions of the effects of the crop insurance? In this paper I employ two models to examine the supply response induced by insurance through changes in variable input use.

The first model considers the case where a producer controls a single input. Such situations of production uncertainty have been widely considered in the literature. The principal insight is that technology matters in determining the impact of uncertainty (MacMinn and Holtmann, Pope and Kramer, Ramaswami). In particular, the property of technology that is relevant is whether an input is risk increasing or risk decreasing. The risk character of an input has been useful in comparative statics analyses. For instance, researchers have used the restrictions on technology implied by a risk increasing or risk decreasing input to predict the impact of public policies on the use of environmentally hazardous inputs like pesticides and herbicides (Antle, Leathers, and Quiggin; Olson and Eidman). The role of insurance has however, not been examined. Because the above studies document risk to be an important consideration in the application of pesticides and herbicides, the impact of insurance may be significant. Consequently, the results of the paper have a bearing on this issue.

The second model considers supply response when a producer can respond with changes in more than one input. The literature on production decisions under production uncertainty is generally confined to the one input case and has little to say about the effects of uncertainty on the choice of input mix and the resultant implications for average supply or its variance. The difficulty is two-fold. First, changes in input mix depend on input substitutability and the distribution of risk increasing and risk decreasing inputs and calls for such knowledge of the production function.[2] Even then, comparative statics are harder because the use of all inputs is simultaneously determined.

Second, the implications for expected supply and other parameters of the output distribution is not straightforward or immediate as this would be in a single input model. For these reasons, the second model employs a dual approach to directly model a producer's choice of output distribution. In the usual primal problem, a producer chooses a vector of inputs to maximize expected utility of profits. However, if the first two moments completely characterize the probability distribution of output (Just and Pope), then the producer's problem can be equivalently posed in terms of choosing a mean and a standard deviation. I show how the equivalence can be utilized to derive comparative statics about the producer's choice of output distribution no matter how many inputs form the input vector.

[2] I owe this point to an anonymous *Journal* referee.

Model: General Assumptions

Farmers are assumed to be risk averse and maximize expected utility of profits where the utility function U is increasing, concave and thrice differentiable. A stochastic technology is described by a twice continuously differentiable production function $q(\mathbf{x}, \boldsymbol{\theta})$ where q is output, $\mathbf{x}$ is a vector of inputs and $\boldsymbol{\theta}$ is a random production shock such that $q_{\boldsymbol{\theta}} > 0$.

An insurance contract consists of an indemnity schedule $I(q)$ and a premium P. Neither the indemnity nor the premium is contingent on a particular choice of input vector. Given a contract $\{I(q), P\}$ a producer chooses input application $\mathbf{x}(I, P)$. When no insurance is purchased or is available, $\mathbf{x}(0, 0)$ denotes the producer's optimal input level. The initial situation is assumed to be one of no insurance. This is compared with a final situation where an insurance contract $\{I, P\}$ is available. The question is, how does the purchase of insurance alter expected output? The answer involves a comparison of $Eq(\mathbf{x}(I, P))$ and $Eq(\mathbf{x}(0, 0))$.

The comparison is carried out for the set of insurance contracts satisfying the following conditions:[3]

(*i*) Feasibility: Given the optimal input responses of farmers, insurance is actuarially fair, i.e., $P = E[I\{q[\mathbf{x}(I, P), \boldsymbol{\theta}]\}]$. Contracts satisfying this condition are feasible. The premium on a feasible contract is equal to the expected level of indemnity.

(*ii*) Differentiability: The indemnity schedule is differentiable everywhere except possibly at a finite number of points. This condition is usually met by real world contracts.[4]

(*iii*) Monotonicity: The indemnity schedule is monotonic decreasing in output (or monotonic increasing in loss), i.e., $I'(q) \leq 0$. Strict monotonicity is not required. In other words, insurance payments are larger (or at least, not smaller) for larger losses. This assumption is also consistent with real-world insurance practice.

Under mild regularity conditions, the set of contracts satisfying the above conditions is nonempty.[5]

Insurance and Supply Response: A Single-Input Model

Here a single input x enters the production function described by $q(x, \boldsymbol{\theta})$. An input is either risk increasing or risk decreasing. If risk increasing, the marginal product $q_x(\cdot, \boldsymbol{\theta})$ is monotonic, increasing in $\boldsymbol{\theta}$ for all positive x. If risk decreasing, $q_x(\cdot, \boldsymbol{\theta})$ is monotonic, decreasing in $\boldsymbol{\theta}$ for all positive x. The terminology derives from the fact that higher levels of input use lead to higher or lower output risk depending on whether $q_{x\boldsymbol{\theta}}$ is positive or negative. As noted later, the optimal application of a risk increasing (decreasing) input under risk averse preferences is always less (more) than the risk-neutral level of input use. In an agricultural context, fertilizers are often risk increasing in their impact on output risk (e.g., Just and Pope) while pesticides and herbicides are risk decreasing (e.g., Antle, Olson and Eidman). A more general interpretation, pursued in Lewis and Nickerson, is to regard $\mathbf{x}$ as expenditures on self-insurance. Then such expenditures are risky if $q_{x\boldsymbol{\theta}} > 0$, but are risk-reducing if $q_{x\boldsymbol{\theta}} < 0$. Many examples of such expenditures are provided in Lewis and Nickerson.

Let $\pi(x, I, P)$ be the producer income as a function of input level and the insurance contract. Then $\pi(x, I, P) = q(x, \boldsymbol{\theta}) - wx + I(q) - P$, where w is the input price and π, q, w, I and P are all normalized with respect to a certain output price. The analysis easily extends to the stochastic price case by regarding q as revenue rather than output. Naturally, the interpretation of the model is affected,[6] but not the results.[7]

[3] Alternatively, the form of the insurance contract could have been endogenously derived (Mirrlees, Holmstrom, Chambers 1989). However, as noted by Hart and Holmstrom, optimal contracts impose little structure on the form of the insurance contract. For instance, the literature is not able to deduce that optimal insurance contracts are increasing in the magnitude of loss.

[4] Consider a U.S. Federal crop insurance contract where

$$I(q) = r(q^* - q) \text{ if } q \leq q^*, r > 0$$
$$= 0 \text{ otherwise.}$$

I is differentiable everywhere except at q^*

[5] Pick a bounded indemnity schedule $I(\cdot)$ which is differentiable and monotone decreasing. The feasible premium consistent with $I(q)$ is the fixed point of the equation: $P = EI(q(\mathbf{x}(I, P), \boldsymbol{\theta}))$. Let $g(y) = EI(q(\mathbf{x}(I, y), \boldsymbol{\theta}))$. g is continuous in y. Let I_1 and I_2 be the lower and upper bounds of the indemnity schedule. If $y \in [I_1, I_2]$, $g(y) \in [I_1, I_2]$. Then by Brouwer's fixed point theorem, there exists a $P \in [I_1, I_2]$ satisfying $P = g(P)$.

[6] First, if $R(x, \boldsymbol{\theta})$ is the gross revenue as a function of input level and a random shock, input x is risk-increasing (or risk-decreasing) if R_x is increasing (or decreasing) in $\boldsymbol{\theta}$ for all x. Second, the insurance program insures revenue and not output.

[7] As this paper considers comparative statics with respect to arbitrary insurance contracts, the model is unaffected by results which show the sensitivity of optimal insurance schemes to output price risk (Ramaswami and Roe).

Complete Insurance

Given an insurance contract, the optimal choice of x is dictated by

(1)
$$\max_x \eta(x, I, P) = EU[q(x, \boldsymbol{\theta}) - wx + \nu(x, \boldsymbol{\theta})]$$

subject to $x \geq >0$
where ν is the payoff from insurance, i.e., $\nu(x, \boldsymbol{\theta}) = I[q(x, \boldsymbol{\theta})] - P$ and η is the expected utility of the producer. With insurance, the change in expected utility as a result of a marginal change in input use is

$$(2) \quad \eta_x(x, I, P) = EU'[\pi(x, I, P)] \cdot \{[1 + I'(q)]q_x(x, \boldsymbol{\theta}) - w\}.$$

An insurance contract is complete if $I'(q) = -1$ for all q, i.e., if insurance fully compensates an incremental loss. For such a contract,

$$\eta_x(x, I, P) = -wEU'(\pi(x, \nu)) < 0$$

for all $x \geq 0$. Thus complete insurance provides no incentives for positive levels of input use.

The rest of the paper assumes that the optimal level of input use is positive. This means insurance is incomplete, i.e., $I'(q) \geq -1$, where the weak inequality is strict at some q.

No-Insurance Case

In the initial situation, $I(q) = P = 0$ for all q, and the incremental change in expected utility due to input use is

(3)
$$\eta_x(x, 0, 0) = EU'(\pi(x, 0, 0))(q_x(x, \boldsymbol{\theta}) - w).$$

Because $\text{cov}(U', q_x) = EU'q_x - EU'Eq_x$, (3) can be rewritten as

$$(4) \quad \eta_x(x, 0, 0) = EU'(\pi(x, 0, 0))(Eq_x(x, \boldsymbol{\theta}) - w) + \text{cov}(U'(\pi(x, 0, 0)), q_x(x, \boldsymbol{\theta})).$$

Dividing by $EU'(\pi(x, 0, 0)Eq_x(x, \boldsymbol{\theta})$, η_x can be expressed, with its sign preserved as a number independent of the units in which output and utility are measured.

$$\eta_x(x, 0, 0)/EU'(\pi(x, 0, 0))Eq_x(x, \boldsymbol{\theta}) = (Eq_x(x, \boldsymbol{\theta}) - w)/Eq_x(x, \boldsymbol{\theta}) + \text{cov}(U'(\pi(x, 0, 0)), q_x(x, \boldsymbol{\theta}))/[EU'(\pi(x, 0, 0))Eq_x(x, \boldsymbol{\theta})]$$

The above can be expressed compactly with the following notation. Let

$$s(x) = (Eq_x(x, \boldsymbol{\theta}) - w)/Eq_x(x, \boldsymbol{\theta})$$

$$\sigma_{MU}(x, I, P) = \sqrt{\text{var}(U'(\pi(x, I, P))}/EU'(\pi(x, I, P))$$

$$\sigma_{MP}(x) = \sqrt{\text{var}(q_x(x, \boldsymbol{\theta}))}/Eq_x(x, \boldsymbol{\theta}) \quad \text{and}$$

$$\rho(x, I, P) = \frac{-\text{cov}(U'(\pi(x, I, P)), q_x(x, \boldsymbol{\theta}))}{\sqrt{\text{var}(q_x(x, \boldsymbol{\theta}))}\sqrt{\text{var}(U'(\pi(x, I, P))}}.$$

s is the fraction of expected marginal product that the producer receives as income after subtracting input costs. σ_{MU} and σ_{MP} are the coefficients of variation of marginal utility and marginal product respectively and ρ is the (negative) correlation between marginal utility and marginal product. Using these definitions,

$$(5) \quad \eta_x(x, 0, 0)/ EU'(\pi(x, 0))Eq_x(x, \boldsymbol{\theta}) = s(x) - \rho(x, 0, 0)\sigma_{MU}(x, 0, 0)\sigma_{MP}(x)$$

(5) expresses the effect of input use on expected utility as the difference between a mean effect (the first term) and a risk effect (the second term). By the first order conditions that characterize the optimum, the two effects are just equal. At the risk-neutral level of input use, the expected marginal product is equal to input cost. This means $s(x)$ is zero and hence so is the risk effect. Thus, whether risk averse farmers use more or less input than the risk-neutral level depends on whether the risk effect is negative or positive.

The sign of the risk effect is determined by ρ, which is the negative of the correlation between marginal utility and marginal product. For concave utility functions, marginal utility is monotonic decreasing in $\boldsymbol{\theta}$. On the other hand, the marginal product is either monotonic increasing or decreasing in $\boldsymbol{\theta}$. It follows that the risk effect is positive if the input is risk increasing and negative if the input is risk decreasing. Consequently, risk averse level of input use is greater (smaller) than the risk-neutral level of input use if the input is risk decreasing (increasing). This result has been noted earlier in MacMinn and Holtmann and in Pope and Kramer.

Risk Reduction and Moral Hazard Effects

A decomposition similar to (5) for η_x (x, I, P) reveals

$$(6)\quad \eta_x(x, I, P)/\mathrm{E}U'(\pi(x, I, P))\mathrm{E}q_x(x, \boldsymbol{\theta}) = s(x) - [\rho(x, I, P)\sigma_{MU}(x, I, P)\sigma_{MP}(x)] - [-\mathrm{E}U'(\pi(x, I, P))\nu_x(x, \boldsymbol{\theta}) /[\mathrm{E}U'(\pi(x, I, P))\mathrm{E}q_x(x, \boldsymbol{\theta})]].$$

Compared to (5), there is an additional moral hazard effect represented by the third term in (6). When a producer buys insurance, a change in input levels, besides affecting output, also affects indemnities. Because an increase in output reduces indemnities, the marginal return to an additional unit of input application is the marginal product of that additional unit less the resulting loss in indemnities; i.e., it is $q_x(1 + I'(q))$ that is less than q_x. The resulting change in expected utility from the loss in indemnities is $\mathrm{E}U'(\pi(x, I, P))q_xI'(q))$, which is the numerator of the third term in (6).

To simplify notation, let $x_i = x(I, P)$ and $x_0 = x(0, 0)$. To compare x_0 with x_i, assume that $\eta(x, 0, 0)$ is concave in x.[8] Then $x_i \gtreqless x_0$ as $\eta_x(x_i, 0, 0) \lesseqgtr 0$ and so checking the latter condition is sufficient. Because $\mathrm{E}U'(\pi(x_i, 0, 0))$ and $\mathrm{E}q_x(x_I, \boldsymbol{\theta})$ are both positive, $\eta_x(x_i, 0, 0)$ is of the same sign as $\eta_x(x_i, 0, 0)/\mathrm{E}U'(\pi(x_i, 0, 0))\mathrm{E}q_x(x, \boldsymbol{\theta})$. From (4),

$$(7)\quad \eta_x(x_i, 0, 0)/ \mathrm{E}U'(\pi(x_i, 0, 0))\mathrm{E}q_x(x_i, \boldsymbol{\theta}) = s(x_i) - \sigma_{MU}(x_i, 0, 0)\sigma_{MP}(x_i)\rho(x_i, 0, 0)$$

But from the first order condition, x_i solves

$$s(x_i) = \sigma_{MU}(x_i, I, P)\sigma_{MP}(x_i)\rho(x_i, I, P) + [- \mathrm{E}U'(\pi(x_i, I, P))\nu_x(x_i, \boldsymbol{\theta})/ [\mathrm{E}U'(\pi(x_i, I, P))\mathrm{E}q_x(x_i, \boldsymbol{\theta})]].$$

Substituting for $s(x_i)$ in (7),

$$(8)\quad \eta_x(x_i, 0, 0)/\mathrm{E}U'(\pi(x_i, 0, 0))\mathrm{E}q_x(x_i, \boldsymbol{\theta}) = [\sigma_{MU}(x_i, I, P)\rho(x_i, I, P) - \sigma_{MU}(x_i, 0, 0)\rho(x_i, 0, 0)] \sigma_{MP}(x_i) + [- \mathrm{E}U'(\pi(x_i, I, P))\nu_x(x_i, \boldsymbol{\theta})/ [\mathrm{E}U'(\pi(x_i, I, P))\mathrm{E}q_x(x_i, \boldsymbol{\theta})]].$$

The first expression in square brackets above is the difference between the risk effect with insurance and the risk effect without insurance, both evaluated at x_i. Because of the following result, the difference between risk effects is called the risk reduction effect.

PROPOSITION 1: *For all constant and decreasing risk averse utility functions and for a feasible insurance contract $\{I, P\}$, (i) $0 < \sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i) < \sigma_{MU}(x_i, 0, 0)\rho(x_i, 0, 0)\sigma_{MP}(x_i)$ if the input is risk-increasing. (ii) $0 > \sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i) > \sigma_{MU}(x_i, 0, 0)\rho(x_i, 0, 0)\sigma_{MP}(x_i)$ if the input is risk-decreasing.*

Proposition 1 is proved in an appendix. As intuition might suggest, proposition 1 asserts that actuarially fair insurance reduces the absolute value of the risk effect of input use. By itself, this leads a producer to adjust input application in the direction of risk-neutral levels, i.e., to increase input use if risk increasing and to decrease input use if risk decreasing.

However, the sign of $\eta_x(x_i, 0, 0)$ also depends on the sign of the moral hazard effect which is always positive since $\nu_x = I'(q)q_x < 0$ for all x and $\boldsymbol{\theta}$. As remarked earlier, insurance reduces the marginal return from an additional unit of input application which, therefore, reduces input use irrespective of whether it is risk reducing or risk increasing.

Because the moral hazard and the risk reduction effects are in the same direction for a risk decreasing input but of opposite directions for a risk increasing input, crop insurance has different impacts in the two cases. When an input is risk decreasing, both the risk reduction and moral hazard effects are positive which makes $\eta_x(x_i, 0, 0) > 0$. When an input is risk increasing, the risk reduction effect is negative but the moral hazard effect is positive, which makes the sign of $\eta_x(x_i, 0, 0)$ indeterminate. The next result is therefore immediate.

PROPOSITION 2: *With all constant and decreasing risk averse utility functions, the impact of actuarially fair crop insurance on input use is (i) to reduce it if the input is risk decreasing and (ii) indeterminate if the input is risk increasing.*

The implications for supply response are straightforward. If the input is risk decreasing, insurance affects its use in a manner that decreases expected output and increases the riskiness of output. However, if it is risk increasing, (expected) agricultural supply and its riskiness may increase or decrease depending on the relative strengths of the moral hazard and risk reduction effects.

A Numerical Simulation

Because the theoretical analysis is inconclusive in the case of a risk increasing input, a simulation experiment is undertaken to obtain insights. Consider a specialization to linear insur-

[8] Because $\eta_{xx}(x, 0, 0) = \mathrm{E}U'(\pi(x, 0, 0)(q_x(x, \boldsymbol{\theta}) - w)^2 + \mathrm{E}U'(\pi(x, 0, 0))q_{xx}(x, \boldsymbol{\theta})$, concavity of the production function together with risk aversion is sufficient for concavity of expected utility in x.

ance schedules, constant risk averse utility functions, and normally distributed output risk, i.e., $U(y) = -\exp(-Ay)$, $q(x, \boldsymbol{\theta}) = \mu(x) + \sigma(x)\boldsymbol{\theta}$ and $I'(q) = -r$, where A is the constant coefficient of absolute risk aversion, $\boldsymbol{\theta}$ is normally distributed with zero mean and unit variance, and $0 \leq r < 1$. x is risk increasing, i.e., $q_{x\boldsymbol{\theta}} = \sigma_x > 0$.

The optimal input choice is found by solving[9]

(9)
$$(1 - r)\mu_x(x_i) - w - (1 - r)^2 A\sigma(x_i)\sigma_x(x_i) = 0.$$

Note that x_i is independent of P Hence we can write $x_i = x(r)$ to denote the functional dependence on r. Similarly $x_0 = x(0)$.

The decomposition into risk reduction and moral hazard effects can be derived as

(10)
$$\eta_x(x(r), 0, 0)/\mathrm{E}U'(\pi(x(r), 0, 0))\mathrm{E}q_x(x(r), \boldsymbol{\theta}) = (r^2 - 2r)A\sigma(x(r))\sigma_x(x(r))/\mu_x(x(r)) + r$$

from which, we derive

(11) $$x(r) \gtreqless x(0) \text{ as } r \lesseqgtr r^*$$

where $r^* = 2 - \mu_x(x(r))/A\sigma(x(r))\sigma_x(x(r))$

that is, insurance increases input use if insurance is "limited"; otherwise input use declines relative to the zero insurance level. r^* is the critical value of insurance such that, if r is less than r^*, input use increases but if r is greater than r^*, input use declines. The result is consistent with intuition as one would expect the moral hazard effect to be relatively stronger at larger levels of insurance.

The critical value r^* is a function of mean and variance responses to input use and of risk aversion. Let $\alpha = \mu_x x/\mu$, $\beta = \sigma_x x/\sigma$, $s = \sigma/\mu$, $R = \mu A$ and $\rho = Rs^2/2$. α and β are the elasticities of output mean and variance with respect to input application. s is the coefficient of variation of output and R is the coefficient of relative risk aversion evaluated at the mean level of output. ρ is a second order approximation to the relative risk premium, i.e., the fraction of mean income that the producer is willing to pay as risk premium (Newbery and Stiglitz, p. 73). Then

(12) $$x(r) \gtreqless x(0) \text{ as } r \lesseqgtr r^* = 2 - \alpha/(2\rho\beta).$$

The critical level of insurance is higher for larger values of variance elasticity and risk premium and for smaller values of the mean elasticity. Note that if the mean elasticity is much larger than the product of variance elasticity and relative risk premium, r^* could be negative. This means insurance would always decrease input use. For instance if $\rho = 0.5$, r^* is negative whenever the mean elasticity is greater than twice the variance elasticity.

Figures 1 and 2 plot the outcome of a numerical experiment. It is assumed that $\mu(x) = x^\alpha$ and $\sigma(x) = x^\beta$. The optimal value of x is plotted against 20 values of r between 0 and 1. The calculations use equation (9) and assumed values of $w = 0.05$, and $A = 0.2$. In figure 1, the elasticities, α and β, are the ones estimated by Just and Pope for a data set on the output response of corn to nitrogen application. Here the mean elasticity is nearly three times the variance elasticity. Consequently, insurance decreases

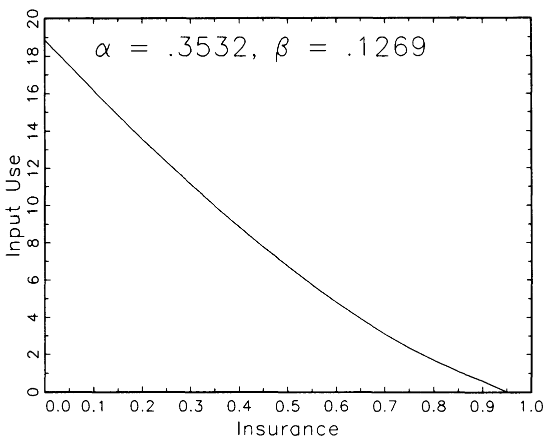

Figure 1. The effect of insurance on the use of a risk-increasing input (β = 0.1269)

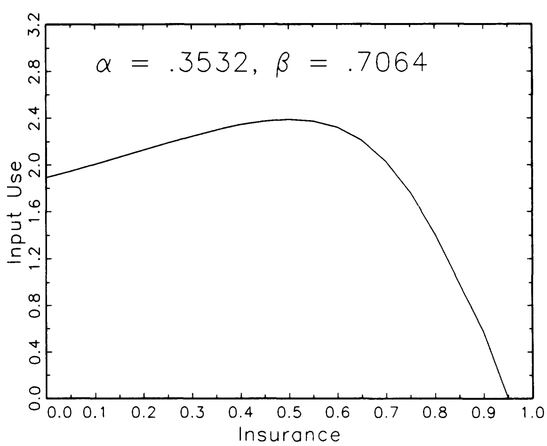

Figure 2. The effect of insurance on the use of a risk-increasing input (β = 0.7064)

[9] The derivation of equations (9) and (10) is available from the author.

input use at all levels of r. Figure 2 is calculated using the same mean elasticity as in figure 1 but the variance elasticity is twice the mean elasticity. As a result, the risk reduction effect is stronger than the moral hazard effect for levels of insurance up to $r^* = 0.7$. Input use is first increasing and then decreasing in r.

Thus, the numerical simulation suggests that the risk reduction effect is stronger (and the impact of insurance on expected supply positive) if insurance is limited, if the producer is highly risk averse, and if the impact of input use on output variability is much larger relative to its impact on expected output.

Insurance and Supply Response: A Multiple-Action Model

For reasons mentioned earlier, a straightforward extension of the single-input case to multiple inputs is not fruitful. In this section, I consider a dual approach to the problem in which a producer directly chooses the parameters of the output distribution. Suppose a production function is of the form $q(\mathbf{x}, \boldsymbol{\theta}) = \mu(\mathbf{x}) + \sigma(\mathbf{x})\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ is a zero mean and unit variance random shock, and $\mathbf{x}$ a vector of n inputs. Such production functions, first proposed by Just and Pope, have been widely used in theoretical and empirical work. Just and Pope showed functions of this type to be less restrictive than the popular alternative of specifying a multiplicative production shock.[10] The primal approach consists of solving the following program.

$$(13)\quad \max_{\mathbf{x}_i} \mathrm{E}U(\mu(\mathbf{x}) + \sigma(\mathbf{x})\boldsymbol{\theta} + I(\mu(\mathbf{x}) + \sigma(\mathbf{x})\boldsymbol{\theta}) - P - \mathbf{wx}),\ i = 1, \ldots, n$$

where $\mathbf{w}$ is a vector of n input prices.

Consider an alternative problem to be solved in two stages. In the first stage, define a cost function $C(\mu, \sigma)$ by

$$(14)\quad C(\mu, \sigma) = \min_{\mathbf{x}_i} \mathbf{wx}, \quad i = 1, \ldots, n$$
$$\text{subject to } \mathbf{x} \in V(\mu, \sigma)$$

where $V(\mu, \sigma) = \{\mathbf{x}: \mu(\mathbf{x}) \geq \mu, \sigma(\mathbf{x}) \leq \sigma\}$. Because $C(\mu, \sigma)$ is like a multi-output cost function, it is well defined and differentiable if the set V is nonempty, closed, and convex (Chambers 1988, ch 7). It is also clear, from standard elementary arguments, that $C_\mu > 0$ and $C_\sigma < 0$.

In the second stage, a producer chooses μ and σ to solve

$$(15)\quad \max_{\mu,\sigma} \psi(\mu, \sigma, I, P) = \mathrm{E}U(Q(\mu, \sigma; \boldsymbol{\theta}) + I(Q(\mu, \sigma)) - P - C(\mu, \sigma))$$

where $Q(\mu, \sigma; \boldsymbol{\theta}) \equiv \mu + \sigma\boldsymbol{\theta}$. Clearly, $Q(\mu, \sigma; \boldsymbol{\theta}) = q(\mathbf{x}, \boldsymbol{\theta})$ whenever $\mathbf{x} \in V(\mu, \sigma)$.

The two-stage problem in (14) and (15) can be considered as a dual to the problem in (13). In the dual of the certainty case, a producer chooses output subject to a cost of choosing of that output. Here in the uncertainty case, a producer chooses an output distribution subject to the costs of choosing that distribution. It remains to be shown that the primal and dual formulations solve the same problem. To see this, it is enough to show that the maximized expected utility from (15) is not less than the expected utility associated with any arbitrary input vector $\hat{\mathbf{x}}$. Let $\hat{\mu} = \mu(\hat{\mathbf{x}})$ and $\hat{\sigma} = \sigma(\hat{\mathbf{x}})$. Also let μ^*, σ^*, and $\mathbf{x}^*$ be the solution to (15) and (14). Then

$$\begin{aligned}
\mathrm{E}U(q(\hat{\mathbf{x}}, \boldsymbol{\theta}) + I(q(\hat{\mathbf{x}}, \boldsymbol{\theta})) - P - \mathbf{w}\hat{\mathbf{x}}) &= \mathrm{E}U(\mu(\hat{\mathbf{x}}) + \sigma(\hat{\mathbf{x}})\boldsymbol{\theta} + I(\mu(\hat{\mathbf{x}}) + \sigma(\hat{\mathbf{x}})\boldsymbol{\theta}) - P - \mathbf{w}\hat{\mathbf{x}})\\
&= \mathrm{E}U(\hat{\mu} + \hat{\sigma}\boldsymbol{\theta} + I(\hat{\mu} + \hat{\sigma}\boldsymbol{\theta}) - P - \mathbf{w}\hat{\mathbf{x}})\\
&\leq \mathrm{E}U(\hat{\mu} + \hat{\sigma}\boldsymbol{\theta} + I(\hat{\mu} + \hat{\sigma}\boldsymbol{\theta}) - P - C(\hat{\mu}, \hat{\sigma}))\\
&= \mathrm{E}U(Q(\hat{\mu}, \hat{\sigma}; \boldsymbol{\theta}) + I(Q(\hat{\mu}, \hat{\sigma}; \boldsymbol{\theta})) - P - C(\hat{\mu}, \hat{\sigma}))\\
&\leq \mathrm{E}U(Q(\mu^*, \sigma^*, \boldsymbol{\theta}) + I(Q(\mu^*, \sigma^*; \boldsymbol{\theta})) - P - C(\mu^*, \sigma^*))\\
&= \mathrm{E}U(q(\mathbf{x}^*, \boldsymbol{\theta}) + I(q(\mathbf{x}^*, \boldsymbol{\theta}) - P - \mathbf{wx}^*)
\end{aligned}$$

Hence the solution from the two stage maximization is identical to the solution from (13).

Let $\mu(I, P)$ and $\mu(0, 0)$ be the mean outputs when producers optimally choose inputs in the presence and absence of insurance respectively. The question again is how purchase of insurance alters mean supply. Solving (15), the mean outputs with and without insurance can be directly compared. The comparative statics are carried out by assuming $\psi(\mu, \sigma, 0, 0)$ to satisfy the sufficient conditions for concavity, namely, $\psi_{\mu\mu} < 0$, $\psi_{\sigma\sigma} < 0$ and $\psi_{\mu\mu}\psi_{\sigma\sigma} - \psi^2_{\mu\sigma} > 0$. In order to ensure strictly positive solutions, it is also assumed that $\lim_{\mu\to 0} C(\mu, \sigma) = 0$ and $\lim_{\sigma\to 0} C(\mu, \sigma) = \infty$.

[10] The dual approach is also available for the multiplicative case. Suppose $q = \mu(\mathbf{x})\boldsymbol{\theta}$. The dual in this case considers the choice of μ alone and the problem reduces to the selection of one variable, which can then be handled by slight modification of the single input case considered earlier (see, for example, Eeckhoudt and Hansen).

No-Insurance Case

In the initial situation, $I(q) = P = 0$ for all q and the marginal changes in expected utility are given by

$$(16)\quad \psi_\mu(\mu, \sigma, 0, 0) = EU'(\pi(\mu, \sigma, 0, 0))(1 - C_\mu(\mu, \sigma))$$

$$(17)\quad \psi_\sigma(\mu, \sigma, 0, 0) = EU'(\pi(\mu, \sigma, 0, 0))(\boldsymbol{\theta} - C\sigma(\mu, \sigma))$$

where $\pi(\mu, \sigma, 0, 0) = Q(\mu, \sigma; \boldsymbol{\theta}) - C(\mu, \sigma)$. Rewriting the above equations,

$$(18)\quad \psi_\mu(\mu, \sigma, 0, 0)/EU'(\pi(\mu, \sigma, 0, 0)) = 1 - \psi_\mu(\mu, \sigma) \text{ and}$$

$$(19)\quad \psi_\sigma(\mu, \sigma, 0, 0)/EU'(\pi(\mu, \sigma, 0, 0)) = -\psi_\sigma(\mu, \sigma) + \operatorname{cov}(U'(\pi(\mu, \sigma, 0, 0)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma, 0, 0))$$

For a risk-neutral producer,

$$(20)\quad \psi_\sigma(\mu, \sigma, 0, 0) = - C_\sigma(\mu, \sigma)[EU'(\pi(\mu, \sigma, 0, 0))] > 0 \text{ for all } \sigma.$$

Hence for a given μ, a risk-neutral producer chooses the maximum possible σ. Let $\sigma_m(\mu) = \max_{\mathbf{x}} \sigma(\mathbf{x})$ subject to $\mu(\mathbf{x}) = \mu$. Substituting in (16), a risk-neutral producer's choice of mean output, μ_n, satisfies

$$(21)\quad \psi_\mu(\mu_n, \sigma_m(\mu_n), 0, 0)/EU'(\pi(\mu_n, \sigma_m(\mu_n), 0, 0)) = (1 - C_\mu(\mu_n, \sigma_m(\mu_n)) = 0$$

Let $\sigma_a(\mu)$ be a function which describes a risk averse producer's choice of σ for a given μ. $\sigma_a(\mu)$ solves

$$(22)\quad \psi_\sigma(\mu, \sigma_a(\mu), 0, 0) = EU'(\pi(\mu, \sigma_a(\mu), 0, 0))(\boldsymbol{\theta} - C_\sigma(\mu, \sigma_a(\mu))) = 0.$$

Because $\psi_{\sigma\sigma} < 0$, (20) and (22) imply $\sigma_a(\mu) < \sigma_m(\mu)$ for all μ. Substituting $\sigma_a(\mu)$ in the first order condition for μ, a risk averse producer's optimal choice of mean output, μ_0, solves

$$(23)\quad \psi_\mu(\mu_0, \sigma_a(\mu_0), 0, 0)/EU'(\pi(\mu_0, \sigma_a(\mu), 0, 0)) = 1 - C_\mu(\mu_0, \sigma_a(\mu_0))$$

A sufficient condition for determining the relation between μ_0 and μ_n is stated below.

PROPOSITION 3: *The expected output of a risk averse producer is greater than (equal to, less than) the expected output of a risk-neutral producer if $C_{\mu\sigma} > (=, <) 0$.*

PROOF: Define $\phi(\mu) \equiv \psi_\mu(\mu, \sigma_a(\mu), 0, 0)$. Then $\phi'(\mu) = \psi_{\mu\mu} + \psi_{\sigma\mu}\, \sigma_{a'}(\mu)$. But applying the implicit function theorem to (22), $\sigma_{a'}(\mu) = - (\psi_{\sigma\mu} /\psi_{\sigma\sigma})$. Substituting, $\phi'(\mu) = (\psi_{\sigma\sigma}\, \psi_{\mu\mu} - \psi^2_{\sigma\mu})/ \psi_{\sigma\sigma} < 0$ by our assumptions on ψ. Since ϕ is decreasing in μ and $\phi(\mu_0) = 0$, $\mu_0 \gtreqless \mu_n$ as $\phi(\mu_n) \gtreqless 0$. Now $\phi(\mu_n)/EU'(\pi(\mu_n, \sigma_a(\mu_n), 0, 0)) = 1 - C_\mu(\mu_n, \sigma_a(\mu_n))$. Using (21), $\phi(\mu_n)/EU'(\pi(\mu_n, \sigma_a(\mu_n), 0, 0)) = C_\mu(\mu_n, \sigma_m(\mu_n)) - C_\mu(\mu_n, \sigma_a(\mu_n))$. But $\sigma_m(\mu) > \sigma_a(\mu)$ for all μ. Hence, $\phi(\mu_n)/ EU'(\pi(\mu_n, \sigma_a(\mu_n), 0, 0)) \gtreqless 0$ if $C_{\mu\sigma} \gtreqless 0$. Because expected marginal utility is strictly positive, $\phi(\mu_n) \gtreqless 0$ if $C_{\mu\sigma} \gtreqless 0$.

Risk Reduction and Moral Hazard Effects

With insurance, the equations analogous to (18) and (19) are

$$(24)\quad \psi_\mu(\mu, \sigma, I, P)/EU'(\pi(\mu, \sigma, I, P)) = 1 - C_\mu(\mu, \sigma) + EU'(\pi(\mu, \sigma, I, P))I'(q)/EU'(\pi(\mu, \sigma, I, P))$$

$$(25)\quad \psi_\sigma(\mu, \sigma, I, P)/EU'(\pi(\mu, \sigma, I, P)) = - C_\sigma(\mu, \sigma) + \operatorname{cov}(U'(\pi(\mu, \sigma, I, P))\,(1 + I'(Q)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma, I, P))$$

where

$$\pi(\mu, \sigma, I, P) = Q(\mu, \sigma; \boldsymbol{\theta}) + I(Q(\mu, \sigma; \boldsymbol{\theta})) - P - C(\mu, \sigma).$$

Let $\sigma_i(\mu)$ be an insured producer's optimal choice of σ as a function of μ. It satisfies

$$(26)\quad -C_\sigma(\mu, \sigma_i(\mu)) + \operatorname{cov}(U'(\pi(\mu, \sigma_i(\mu), I, P))(1 + I'(Q)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma_i(\mu), I, P)) = 0$$

Denote $\mu_i \equiv \mu(I, P)$. μ_i solves

$$(27)\quad 1 - C_\mu(\mu_i, \sigma_i(\mu_i)) + EU'(\pi(\mu_i, \sigma_i(\mu_i), I, P))I'(Q)/EU'(\pi(\mu_i, \sigma_i(\mu_i), I, P)) = 0.$$

Comparing the first order conditions for μ with and without insurance, that is, equations (23) and (27), note that with σ held constant, insurance decreases expected output because of the third term in (27), which is nothing but the moral hazard effect. However, insurance changes the choice of σ and this also matters in determining the final impact of insurance.

From the proof of Proposition 3, we know ϕ is decreasing in μ and $\phi(\mu_0) = 0$. Then, $\mu_i \gtreqless \mu_0$ as $\phi(\mu_i) \lesseqgtr 0$. Now $\phi(\mu_i)/\mathrm{E}U'(\pi(\mu_i, \sigma_a(\mu_i), 0, 0)) = 1 - C_\mu(\mu_i, \sigma_a(\mu_i))$. Using (27),

$$(28)\quad \phi(\mu_i)/\mathrm{E}U'(\pi(\mu_i, \sigma_a(\mu_i), 0, 0)) = C_\mu(\mu_i, \sigma_i(\mu_i)) - C_\mu(\mu_i, \sigma_a(\mu_i)) - \mathrm{E}U'(\pi(\mu_i, \sigma_i(\mu_i), I, P))I'(q)/\mathrm{E}U'(\pi(\mu_i, \sigma_i(\mu_i), I, P)).$$

The last term, which is the moral hazard effect, is positive and by itself (i.e., ignoring the first two terms) reduces mean output. The difference between the first two terms is the risk reduction effect. The sign and magnitude of the risk reduction effect depends on the difference between $\sigma_i(\mu_i)$ and $\sigma_a(\mu_i)$ and on the sign of the cross derivative $C_{\mu\sigma}$.

PROPOSITION 4: *For all constant and decreasing risk averse utility functions and for convex insurance schedules, $\sigma_i(\mu) > \sigma_a(\mu)$ for all μ and*

$$(29)\quad C_\mu(\mu_i, \sigma_i(\mu_i)) - C_\mu(\mu_i, \sigma_a(\mu_i)) > 0 \text{ if } C_{\mu\sigma} > 0 \text{ and}$$

$$(30)\quad C_\mu(\mu_i, \sigma_i(\mu_i)) - C_\mu(\mu_i, \sigma_a(\mu_i)) < 0 \text{ if } C_{\mu\sigma} < 0.$$

Proposition 4 is proved in the appendix. Its implication is that insurance decreases expected output if $C_{\mu\sigma} > 0$. On the other hand, if $C_{\mu\sigma} < 0$, the moral hazard and risk reduction effects run in opposite directions and the net effect is indeterminate. Note that the class of convex insurance schemes includes real-world insurance schedules which are piecewise linear.

Concluding Remarks

I have decomposed the impact of crop insurance on variable input use into a moral hazard and a risk reduction effect. The principal insight is that insurance changes the marginal costs of input use in two ways. First, insurance reduces risk and therefore reduces the wedge between expected marginal product and input price due to risk aversion. This is the risk reduction effect which leads risk averse decisions toward risk-neutral levels. Mean output increases or decreases depending on the underlying technology. Second, insurance reduces the marginal productivity of all inputs, as an increase in output is always accompanied by a decrease in expected insurance indemnities. This is the moral hazard effect which reduces the use of all inputs and decreases mean output. In earlier work (Ahsan, Ali and Kurian, Nelson and Loehman), the moral hazard effect is absent because insurance indemnities are contingent on input levels.

If the risk reduction effect is the sole impact, mean output adjusts in the direction of risk-neutral levels. In the presence of moral hazard, however, this need not necessarily happen. The numerical simulation suggests that the risk reduction effect dominates moral hazard only if risk aversion is high and if the impact of input use on output risk is large relative to its impact on expected output.

Future work could extend these results to allow activity choice. It is often suggested that crop insurance leads to cultivation of riskier crops, adoption of riskier production techniques, and use of marginal, high-risk farm lands (see for example, Todd, Gardner, and Kramer). In other words, crop insurance could increase expected supply by promoting specialization in the riskier activities. Such argument can be examined in the dual formulation proposed here by interpreting $\mathbf{x}$ as a vector of all producer decisions (including activity and input choices) and q as aggregate revenue from all activities. Results obtained here suggest that insurance may not always increase expected supply. This would not be surprising, since it seems reasonable to suppose that moral hazard in input choice might limit the specialization achievable by insurance. Risk averse farmers would specialize in the riskier activity only if they were to be offered sufficient insurance. But that may reduce input use in the riskier activity due to the moral hazard effect. If input use decreases, expected returns from the riskier activity also decrease, which limits specialization in the riskier activity.

These findings point to a trade-off in the design of insurance. In many developing countries, agricultural insurance programs are expected to increase agricultural production by inducing the use of riskier inputs and technologies. The implicit belief is that insurance programs have strong risk-reduction effects. However, this is true only in the absence of moral hazard. In the real world, insurers find it infeasible to monitor all production activities of farmers. A badly designed insurance program could have moral hazard effects which completely erode the incentives to produce higher output due to risk reduction. One way to control moral hazard is to limit the amount of insurance through higher deductibles or lower price elections. Unfortunately, lower amounts of insurance also have correspondingly weaker risk reduction effects. Similarly, while insurance schemes contingent on area rather than individual yield can reduce and even eliminate moral hazard, their risk reduction effects on individual producers are also smaller. The challenge, therefore, is to design and administer insurance programs which reduce moral hazard while preserving risk reduction effects. In evaluating alternative insurance schemes, simulation exercises could be used to assess the trade-off between risk reduction and moral hazard effects.

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Appendix

Proof of Proposition 1

Substituting for σ_{MU}, σ_{MP} and ρ,

$$\text{(A1)}\quad \sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i) = -\text{cov}(U'(\pi(x_i, I, P)), q_x(x_i, \theta))/[EU'(\pi(x_i,I,P))E_{qx}(x_i, \theta)].$$

The sign of (A1) is opposite to the sign of $\text{cov}(U'(\pi(x_i, I, P)), q_x(x_i, \theta))$. U' is decreasing in θ since $dU'/dq\theta = U'(\pi(x, I, P))(1 + I'(q))q_\theta < 0$ given the assumptions $q_\theta > 0$, $I'(q) \geq -1$ and $U' < 0$. On the other hand q_x is monotonic increasing or decreasing in θ depending on the technology. Thus $\sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i)$ is positive if the input is risk increasing and negative if the input is risk decreasing. This proves the first inequality in (*i*) and (*ii*).

Next, consider the relationship between $\sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i)$ and $\sigma_{MU}(x_i, 0, 0)\rho(x_i, 0, 0)\sigma_{MP}(x_i)$. Let $D = \sigma_{MU}(x_i, 0, 0)\rho(x_i, 0, 0)\sigma_{MP}(x_i) - \sigma_{MU}(x_i, I, P)\rho(x_i, I, P)\sigma_{MP}(x_i)$. The proof consists in showing D to be positive (or negative) if $\mathbf{x}$ is risk-increasing (or risk-decreasing). Substituting for σ_{MU}, σ_{MP} and ρ,

$$D = -\text{cov}(U'(\pi(x_i, 0, 0)), q_x(x_i, \theta))/EU'(\pi(x_i, 0, 0)E_{qx}(x_i, \theta) + \text{cov}(U'(\pi(x_i, I, P)), q_x(x_i, \theta))/EU'(\pi(x_i, I, P))E_{qx}(x_i, \theta).$$

Let $\pi(x_i, I, P) = \pi_i$ and $\pi(x_i, 0, 0) = \pi_0$. Multiplying throughout by E_{qx},

$$\begin{aligned} D[E_{qx}(x_i, \theta)] &= \text{cov}(U'(\pi_i), q_x(x_i, \theta))/EU'(\pi_i) - \text{cov}(U'(\pi_0), q_x(x_i, \theta))/EU'(\pi_0) \\ &= (EU'(\pi_i)q_x(x_i, \theta) - EU'(\pi_i)E_{qx}(x_i, \theta))/EU'(\pi_i) \\ &\quad - (EU'(\pi_0)q_x(x_i, \theta) - EU'(\pi_0)E_{qx}(x_i, \theta))/EU'(\pi_0) = EU'(\pi_i)q_x(x_i, \theta)/EU'(\pi_i) \\ &\quad - EU'(\pi_0)q_x(x_i, \theta)/EU'(\pi_0) \\ &= E\{U'(\pi_i)/EU'(\pi_i) - U'(\pi_0)/EU'(\pi_0)\}q_x(x_i, \theta) \\ &= E(G(x_i, \theta)q_x(x_i, \theta)) \end{aligned}$$

where $G = U'(\pi_i)/EU'(\pi_i) - U'(\pi)/EU'(\pi)$. But $EG(x_i, \theta) = 0$. So

$$\text{(A2)}\quad D[E_{qx}(x_i, \theta)] = \text{cov}(G(x_i, \theta), q_x(x_i, \theta))$$

The following result, proved later, is useful in signing the above covariance.

LEMMA: *For all non-increasing risk averse utility functions, there exists a* θ^* *such that* $G(x_i, \theta) > 0$ *for all* $\theta > \theta^*$ *and* $G(x_i, \theta) < 0$ *for all* $\theta < \theta^*$.

(A2) can now be signed using the lemma and the methods of Hildreth and Tesfatsion. If F is the cumulative density of θ with support $[\theta_m, \theta_M]$, (A2) can be written as

$$\text{cov}(G(x_i, \theta), q_x(x_i, \theta)) = \int_{\theta_m}^{\theta^*} G(\mathbf{x}_i, \theta)q_x(\mathbf{x}_i, \theta)dF(\theta) + \int_{\theta^*}^{\theta_M} G(x_i, \theta)q_x(x_i, \theta)dF(\theta).$$

Suppose $q_{x\theta} > 0$. Then for all $\theta < \theta^*$, $q_x(x_i, \theta) < q_x(x_i, \theta^*)$. But also for these values for θ, $G(x_i, \theta) < 0$ (by lemma). Hence

$$\text{(A3)}\quad \int_{\theta_m}^{\theta^*} G(x_i, \theta)q_x(x_i, \theta)dF(\theta) > q_x(x_i, \theta^*)\int_{\theta_m}^{\theta^*} G(\mathbf{x}_i, \theta)dF(\theta).$$

For all $\theta > \theta^*$, $q_x(\mathbf{x}_i, \theta) > q_x(\mathbf{x}_i, \theta^*)$. But also for these values for θ, $G(\mathbf{x}_i, \theta) > 0$ (by lemma). Hence

$$\text{(A4)}\quad \int_{\theta^*}^{\theta_M} G(x_i, \theta)q_x(x_i, \theta)dF(\theta) > q_x(x_i, \theta^*)\int_{\theta^*}^{\theta_M} G(x_i, \theta)dF(\theta)$$

Combining (A3) and (A4),

$$\text{cov}(G(x_i, \theta), q_x(x_i, \theta)) > q_x(x_i, \theta^*)\int_{\theta_m}^{\theta_M} G(x_i, \theta)dF(\theta) = q_x(x_i, \theta^*)EG(x_i, \theta) = 0.$$

This proves the second inequality in (*i*). The second inequality in (*ii*) is proved similarly as well.

Turning to the proof of the lemma, note that

$$G(x_i, \theta) \gtreqless 0 \text{ as } U'(\pi_i)/U'(\pi_0) \gtreqless EU'(\pi_i)/EU'(\pi_0).$$

Let $T(\theta) = U'(\pi_i)/U'(\pi_0)$. Now $\pi_i = \pi_0 + \nu(x_i, \theta)$ where $\nu = I(q) - P$. Since $\{I, P\}$ is a feasible contract, $P = EI(q(x_i, \theta))$ or $E\nu(x_i, \theta) = 0$. Further, as I is decreasing in θ, ν is also decreasing in θ. Thus there exists an unique $\hat{\theta}$ such that $I(q(x_i, \theta)) \gtreqless P$ or $\nu(x_i, \theta) > 0$ for all $\theta \lesseqgtr \hat{\theta}$. From the monotonicity of U' it follows that

$$\text{(A5)}\quad T(\theta) > 1 \text{ for all } \theta > \hat{\theta} \text{ and } T(\theta) \leq 1 \text{ for all } \theta \leq \hat{\theta}.$$

Let $C = EU'(\pi_i)/EU'(\pi_0)$. Because $E\nu(x_i, \theta) = 0$ and ν is decreasing in θ, π_0 is riskier than π_i by a mean preserving spread. But if risk aversion is non-increasing, marginal utility is convex. It follows (Rothschild and Stiglitz) that $EU'(\pi_i) < EU'(\pi_0)$ and $C < 1$. Let θ^* be such that $T(\theta^*) = C$. θ^* exists because $G(x_i, \theta) \gtreqless 0$ as $T(\theta) \gtreqless C$ and $EG(x_i, \theta) = 0$. Due to (A5) and the fact that $C < 1$, θ^* must be less than $\hat{\theta}$. But T is increasing in θ in this range. Hence θ^* is unique.

$$\begin{aligned} \text{Now, } T'(\theta) &= \frac{U'(\pi_0)U''(\pi_i)\pi_i'(\theta) - U'(\pi_i)U''(\pi_0)\pi_0'(\theta)}{U'(\pi_0)^2} \\ &= \frac{U'(\pi_0)U''(\pi_i)q_\theta(1 + I'(q)) - U'(\pi_i)U''(\pi_0)q_\theta}{U'(\pi_0)^2} \\ &= \frac{\{U'(\pi_0)U''(\pi_i) - U'(\pi_i)U''(\pi_0)\}q_\theta}{U'(\pi_0)^2} + \frac{U'(\pi_0)U''(\pi_i)I'(q)q_\theta}{U'(\pi_0)^2} \end{aligned}$$

$$\text{(A6)}\quad T'(\theta) = \left[\frac{U'(\pi_i)q_\theta}{U'(\pi_0)}\right](A(\pi_0) - A(\pi_i)) + \left[\frac{U''(\pi_i)q_\theta I'(q))}{U'(\pi_0)}\right]$$

where $A(\pi)$ is the coefficient of absolute risk aversion evaluated at π. The second term in the above expression is positive since $U'' < 0$ and $I'(q) \leq 0$. The first term is non-negative for non-increasing risk averse utility functions whenever $\pi_i > \pi_0$, i.e., $\boldsymbol{\theta} < \hat{\boldsymbol{\theta}}$. Thus T is increasing in $\boldsymbol{\theta}$ for all $\boldsymbol{\theta} \leq \hat{\boldsymbol{\theta}}$.

$$\text{Because } T'(\boldsymbol{\theta}) > 0 \text{ for all } \boldsymbol{\theta} < \hat{\boldsymbol{\theta}},$$
$$T(\boldsymbol{\theta}) < C \text{ for all } \boldsymbol{\theta} < \boldsymbol{\theta}^* \text{ and}$$
$$T(\boldsymbol{\theta}) \geq C \text{ for all } \boldsymbol{\theta} \in [\boldsymbol{\theta}^*, \hat{\boldsymbol{\theta}}].$$

But from (A5), $T(\boldsymbol{\theta}) > 1 > C$ for all $\boldsymbol{\theta} > \hat{\boldsymbol{\theta}}$. Hence, $\boldsymbol{\theta}^*$ satisfies

$$T(\boldsymbol{\theta}) < C \text{ for all } \boldsymbol{\theta} < \boldsymbol{\theta}^* \text{ and}$$
$$T(\boldsymbol{\theta}) \geq C \text{ for all } \boldsymbol{\theta} > \theta^*.$$

This proves the lemma.

PROOF OF PROPOSITION 4: (29) and (30) follow trivially once it is shown that $\sigma_i(\mu) > \sigma_a(\mu)$. For notational simplicity, the functional dependence of σ_a and σ_i on μ will henceforth be suppressed. Because $\psi_\sigma(\mu, \sigma_a, 0, 0) = 0$ and $\psi_{\sigma\sigma}(\mu, \sigma, 0, 0) < 0$, $\sigma_i > \sigma_a$ if $\psi_\sigma(\mu, \sigma_i, 0, 0) < 0$. Now $\psi_\sigma(\mu, \sigma_i, 0, 0)/EU'(\pi(\mu, \sigma_i, 0, 0)) = -C(\mu, \sigma_i)) + \mathrm{cov}(U'(\pi(\mu, \sigma_i, 0, 0)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma_i, 0, 0))$.
Substituting for $C(\mu, \sigma_i)$ (from (26)),

$$\begin{aligned}
\text{(A7)} \quad \psi_\sigma(\mu, \sigma_i, 0, 0)/EU'(\pi(\mu, \sigma_i, 0, 0)) = {} & -\mathrm{cov}(U'(\pi(\mu, \sigma_i, I, P))(1 \\
& + I'(q)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma, I, PP)) \\
& + \mathrm{cov}(U'(\pi(\mu, \sigma_i, 0, 0)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma_i, 0, 0)) \\
= {} & \mathrm{cov}(U'(\pi(\mu, \sigma_i, 0, 0)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma_i, 0, 0)) \\
& - \mathrm{cov}(U'(\pi(\mu, \sigma_i, I, P)), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma, I, PP)) \\
& - \mathrm{cov}(U'(\pi(\mu, \sigma_i, I, P))I'(q), \boldsymbol{\theta})/EU'(\pi(\mu, \sigma, I, PP)).
\end{aligned}$$

The difference between the first two terms can be shown to be negative by the methods used to prove part (*i*) of proposition 1. The details are not repeated here. What remains to be shown is that the third term is also negative. This term is negative if $U'(\pi(\mu, \sigma_i, I, P)I'(q)$ is increasing in $\boldsymbol{\theta}$.

Consider $\boldsymbol{\theta}_1 > \boldsymbol{\theta}_2$. Let U_1', I_1' and U_2', I_2' be the marginal utility and slope of the insurance schedule evaluated at $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ respectively. Then we need to show $U_1'I_1' > U_2'I_2'$. Marginal utility U' is decreasing in θ since $\partial U'/\partial\boldsymbol{\theta} = U'(1 + I')\sigma < 0$. Consequently, $U_1' < U_2'$. Since $I_2' < 0$, $U_1'I_2' > U_2'I_2'$. On the other hand, $I_1' > I_2'$ by the convexity of I. But $U_1' > 0$ and so $U_1'I_1' > U_1'I_2'$. Combining the inequalities, $U_1'I_1' > U_2'I_2'$.